

PARTON DISTRIBUTIONS FOR THE PION IN A CHIRAL QUARK MODEL*

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Parton distributions for the pion are studied in a chiral quark model characterized by a quark propagator for which a spectral representation is assumed. Electromagnetic and chiral symmetry constraints are imposed through the relevant Ward-Takahashi identities for flavoured vertex functions. Finiteness of the theory, requires the spectral function to be non-positive definite. Straightforward calculation yields the result that the pion structure function becomes one in the chiral limit, regardless of the details of the spectral function. LO and NLO evolution provide a satisfactory description of phenomenological parameterizations of the valence distribution functions but fails to describe gluon and sea distributions.

1 Introduction

Deep inelastic scattering (DIS) is regarded as one of the traditional ways to unveil the quark substructure of hadrons ¹. The hadronic tensor for electroproduction on a (unpolarized) hadron is defined as

$$W_{\mu\nu}(p, q) = \frac{1}{4\pi} \int d^4\xi e^{iq \cdot \xi} \langle p | [J_\mu^{\text{em}}(\xi), J_\nu^{\text{em}}(0)] | p \rangle \quad (1)$$

where p is the hadron momentum, $J_\mu^{\text{em}}(\xi) = \bar{q}(\xi) \hat{Q} \gamma_\mu q(\xi)$ the electromagnetic current with $\hat{Q} = \text{diag}(e_u, e_d, e_s)$ the quark charge matrix and q the photon momentum transfer. Introducing the relativistically invariant Euclidean momentum transfer $Q^2 = -q^2$ and the Bjorken variable $x = Q^2/2p \cdot q$, in the Bjorken limit ($Q^2 \rightarrow \infty$ with x fixed), Eq.(1) can be expressed in terms of the quark and antiquark distribution functions, $q_i(x)$ and $\bar{q}_i(x)$

$$W_{\mu\nu} \xrightarrow{\text{Bj}} F(x) \left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} - \frac{1}{q^2} \left(p_\mu - \frac{q_\mu}{2x} \right) \left(p_\nu - \frac{q_\nu}{2x} \right) \right] \quad (2)$$

with $F(x) = \sum_i e_i^2 [q_i(x) + \bar{q}_i(x)]/2$ as a consequence of Lorentz gauge invariance, scaling and the underlying spin 1/2 nature of quarks. Logarithmic scaling violations due to perturbative QCD radiative corrections ², relate in

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a linear fashion the structure functions at a given reference scale, Q_0^2 , to the scale of interest, Q^2 ,

$$F(x, Q^2) = U(Q^2, Q_0^2)F(x, Q_0^2). \quad (3)$$

Here, $U(Q^2, Q_0^2)$ is a linear matrix operator, fulfilling renormalization group properties $U(Q_1^2, Q_2^2)U(Q_2^2, Q_3^2) = U(Q_1^2, Q_3^2)$ and $U(Q^2, Q^2) = 1$ and which can be evaluated for high Q^2 . Thus, one is left with the calculation of structure functions at a certain scale Q_0 (the so called initial condition, $F(x, Q_0^2)$) and the assumption that the scale is high enough to make perturbative matching meaningful.

In spite of the wealth of experimental and theoretical information^{3,4,5,6,7}, first principle calculations of the initial condition, $F(x, Q_0^2)$, remain, so far, elusive. Thus, theoretical quark models of hadrons and quark-hadron duality assumptions are used. This obviously raises embarrassing questions on confinement, because the intermediate states in the commutator of Eq.(1) should be physical hadron states and, instead, one uses the virtual Compton amplitude⁹

$$T_{\mu\nu}(p, q) = i \int d^4\xi e^{iq \cdot \xi} \langle p | T \{ J_\mu^{\text{em}}(\xi) J_\nu^{\text{em}}(0) \} | p \rangle \quad (4)$$

with $W_{\mu\nu} = 1/(2\pi)\text{Im}T_{\mu\nu}$. Introducing light-cone (LC) coordinates $x = (x^+, x^-, \vec{x}_\perp)$ with $x^\pm = x^0 \pm x^3$ one gets the following expression for the initial condition

$$F(x, Q_0^2) = -\frac{i}{4\pi} \int \frac{dk^- d^2\vec{k}_\perp}{(2\pi)^3} \text{tr} \left[\hat{Q}^2 \gamma^+ \chi(p, k) \right] \Big|_{k^+ = p^+ = mx} \quad (5)$$

where the forward quark-target scattering amplitude is defined

$$\chi(p, k) = -i \int d^4\xi e^{i\xi \cdot k} \langle p | T \{ q(\xi) \bar{q}(0) \} | p \rangle \quad (6)$$

$\chi(p, k)$ corresponds to the unamputated vertex. Eq.(5) holds under the assumption of scaling and finiteness in the Bjorken limit.

Among the hadrons, pseudoscalar mesons and, in particular the pion, appear as the simplest and best understood states from a theoretical viewpoint, since most of its properties can be explained in terms of spontaneous chiral symmetry breaking. Its bound state properties are not expected to depend crucially on confinement, at least for small excitation energies. Furthermore, the pion structure functions have been deduced from phenomenological QCD studies^{5,7} and there also exist lattice calculations of the first few moments¹⁰. Thus, we do not expect to achieve a better theoretical understanding of a DIS process than in the case of the pion. Hence, structure functions of

the pion have been computed several times in different quark-loop models ^{11,12,13,14,15,16,17,18,19}. The issue of implementing gauge invariance and hence proper normalization of $F(x, Q_0^2)$ is problematic and therefore requires some special care.

2 Quark propagator and Ward-Takahashi identities

I present here another model ²⁰ for pion structure based in a spectral representation of the quark propagator and chiral Ward-Takahashi identities (WTI) for the flavoured vertex functions, deduced from conservation of the vector current (CVC) and partial conservation of the axial current (PCAC)

$$J_V^{\mu,a}(x) = \bar{q}(x)\gamma^\mu \frac{\tau_a}{2} q(x) \quad \partial_\mu J_V^{\mu,a}(x) = 0 \quad (7)$$

$$J_A^{\mu,a}(x) = \bar{q}(x)\gamma^\mu \gamma_5 \frac{\tau_a}{2} q(x) \quad \partial_\mu J_A^{\mu,a}(x) = \bar{M}_0 \bar{q}(x) i\tau_a \gamma_5 q(x) \quad (8)$$

with \bar{M}_0 the average up and down current quark masses.

We assume a spectral representation for the quark propagator

$$S(p) = \int dw \frac{\rho(w)}{\not{p} - w} = -i \int d^4x e^{ip \cdot x} \langle 0 | T \{ q(x) \bar{q}(0) \} | 0 \rangle \quad (9)$$

The requirement of a physical Hilbert space would imply the positivity of the spectral function $\rho(w)$, but we are not assuming this here. Predictions based on Eq. (9) require a specific form of $\rho(w)$ as input, but it turns out ²⁰ that in the chiral limit, $\bar{M}_0 \rightarrow 0$ in Eq. (8), many properties of interest are independent of $\rho(w)$. Thus we will focus on those observables and consider the chiral limit. The vector and axial unamputated vertex functions satisfy the WTI

$$(p' - p)_\mu S(p') \Gamma_V^{\mu,a}(p', p) S(p) = \frac{\tau_a}{2} S(p) - S(p') \frac{\tau_a}{2} \quad (10)$$

$$(p' - p)_\mu S(p') \Gamma_A^{\mu,a}(p', p) S(p) = S(p') \frac{\tau_a}{2} \gamma_5 + \gamma_5 \frac{\tau_a}{2} S(p) \quad (11)$$

The gauge technique, in the form introduced in QED ²¹ consists of writing a *solution* for the vector and axial vertices in the form

$$S(p') \Gamma_V^{\mu,a}(p', p) S(p) = \int dw \rho(w) \frac{1}{\not{p}' - w} \gamma^\mu \frac{\tau_a}{2} \frac{1}{\not{p} - w} \quad (12)$$

$$S(p') \Gamma_A^{\mu,a}(p', p) S(p) = \int dw \rho(w) \frac{1}{\not{p}' - w} \left(\gamma^\mu - \frac{2wq^\mu}{q^2} \right) \gamma_5 \frac{\tau_a}{2} \frac{1}{\not{p} - w} \quad (13)$$

with $q = p' - p$. An interesting consequence of the axial Ward identity (13) is the presence of a massless pseudoscalar pole, which is identified with the pion.

3 Properties of the spectral function

Some properties on the spectral function $\rho(w)$ can be derived by studying specific processes and comparing with known QCD results. For instance, vacuum polarization can be computed closing the vector-quark-quark vertex in Eq. (12)

$$i \int d^4x e^{-ip \cdot x} \langle 0 | T \{ J_\mu^{\text{em}}(x) J_\nu^{\text{em}}(0) \} | 0 \rangle = \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) \Pi^{\text{em}}(p^2) \quad (14)$$

The $e^+e^- \rightarrow \text{hadrons}$ cross-section is proportional to the imaginary part of the vacuum charge polarization operator. Asymptotically, one has

$$\sigma(e^+e^- \rightarrow \text{hadrons}) \rightarrow \frac{4\pi\alpha^2}{3s} \left(\sum_i e_i^2 \right) \int dw \rho(w) \quad (15)$$

Thus, the proper QCD asymptotic result is obtained if

$$\int dw \rho(w) = 1 \quad (16)$$

This condition is equivalent to impose that $\lim_{p \rightarrow \infty} pS(p) = 1$.

The weak pion decay constant can be computed from the axial-axial correlation function, since

$$\langle 0 | J_A^{\mu a}(x) | \pi_b(p) \rangle = i f_\pi p_\mu \delta_{a,b} e^{ip \cdot x} \quad (17)$$

and then, inserting a complete set of eigenstates between the currents,

$$\int d^4x e^{-ip \cdot x} \langle 0 | T \{ J_A^{\mu a}(x) J_A^{\nu b}(0) \} | 0 \rangle = i f_\pi^2 \delta_{ab} \frac{p^\mu p^\nu}{p^2} + \dots \quad (18)$$

where the dots indicate regular pieces in the limit $p^2 \rightarrow 0$. Again, closing the axial-quark-quark vertex in Eq. (13) to make a quark loop, and going to the pion pole, $p^2 \rightarrow 0$, one gets

$$f_\pi^2 = \frac{4N_c}{(4\pi)^2} \int dw \rho(w) \frac{1}{i} \int \frac{d^4p}{(2\pi)^4} \frac{w^2}{(p^2 - w^2)^2} \quad (19)$$

The momentum integral is logarithmically divergent. Thus, if

$$\int dw w^2 \rho(w) = 0 \quad \text{then} \quad f_\pi^2 = \frac{4N_c}{(4\pi)^2} \int dw w^2 (-\log w^2) \rho(w) \quad (20)$$

As we see, $\rho(w)$ cannot be a positive definite function, regardless of its support. f_π^2 is positive if $\rho(w)$ is positive around $w = 0$. Notice also that the first condition of Eq. (20) implies that the value of f_π does not depend on any

particular scale used to make the logarithm dimensionless. If $\rho(w)$ had only one zero, the scale of f_π would be set by it.

A direct application of the previous conditions (16) and (20) can be found in the calculation of the pion electromagnetic form factor $F_\pi^{\text{em}}(q^2)$ which complete evaluation requires an explicitly knowledge of the spectral function $\rho(w)$. Besides a proper normalization, $F_\pi^{\text{em}}(0) = 1$, the mean squared radius becomes

$$\langle r_\pi^2 \rangle = -\frac{1}{6} \frac{dF}{dq^2} \Big|_{q^2=0} = \frac{N_c}{4\pi^2 f_\pi^2} \int dw \rho(w) \quad (21)$$

which is the value obtained using the unregularized quark loop if the normalization condition (16) is used. Thus, the pion has a finite size in this framework.

The amplitude for the (anomalous) neutral pion decay $\pi^0(p) \rightarrow \gamma(p_1, \mu) + \gamma(p_2, \nu)$ can also be computed from the triangle diagram yielding for on-shell massless pions $p_1^2 = p_2^2 = 0$, the result

$$M_{\mu\nu}(p_1, p_2) = \frac{N_c}{12\pi^2 f_\pi} \epsilon_{\mu\nu\alpha\beta} p_2^\alpha p_1^\beta \int dw \rho(w) \quad (22)$$

which, again, coincides with the standard one if Eq.(16) is used.

4 Pion Structure Function

An advantage of our method is that calculations can be directly done in Minkowski space and hence we may directly profit from LC coordinates. For the evaluation of the structure function of the pion the relevant vertex is constructed from two axial currents and is defined

$$(2\pi)^4 \delta(p' + q' - p - q) \chi_{AA}^{\mu, a; \nu, b}(p', q'; p, q) \\ = i \int d^4x d^4x' d^4y d^4y' \langle 0 | T \left\{ J_A^{\mu, a}(x) J_A^{\nu, b}(x') q(y) \bar{q}(y') \right\} | 0 \rangle e^{i[q' \cdot x' + p' \cdot y' - q \cdot x - p \cdot y]} \quad (23)$$

and fulfills the WTl

$$iq^\mu \chi_{AA}^{\mu, a; \nu, b}(p', q'; p, q) = \epsilon_{bac} S(p') \Gamma_V^{\nu, c}(p', p) S(p) \\ + \frac{\tau_a}{2} \gamma_5 S(p' - q) \Gamma_A^{\nu b}(p' - q, p) S(p) + S(p') \Gamma_A^{\nu b}(p', p + q) S(p + q) \frac{\tau_a}{2} \gamma_5 \quad (24)$$

Up to transverse pieces one gets the solution

$$\chi_{AA}^{\mu, a; \nu, b}(p', q'; p, q) = \int dw \rho(w) \frac{1}{\not{p}' - w} \left\{ \frac{w q'^\nu q^\mu}{q'^2 q^2} \delta_{ab} \right. \\ \left. + \left(\gamma_\nu + \frac{2w q'^2}{q'^2} \right) \gamma_5 \frac{\tau_b}{2} \frac{1}{\not{p} + \not{q} - w} \left(\gamma_\mu - \frac{2w q^\mu}{q^2} \right) \gamma_5 \frac{\tau_a}{2} + \text{crossed} \right\} \frac{1}{\not{p}' - w} \quad (25)$$

At the pion poles, $q^2, q'^2 \rightarrow 0$, we get the $\pi q \rightarrow \pi q$ unamputated amplitude in the forward direction ($q = q'$)

$$\chi_{AA}^{\mu,a;\nu,b}(p, q; p, q) \rightarrow \frac{q^\nu q^\mu}{q^4} f_\pi^2 \chi^{ba}(p, q) \quad (26)$$

and direct use of Eq.(5) together with the conditions (16) and (20) yields the initial condition for the quark distributions in π^+ ,

$$u_\pi(x, Q_0^2) = \bar{d}_\pi(1-x, Q_0^2) = \theta(x)\theta(1-x) \quad (27)$$

This result has been derived in the chiral limit, is independent of the spectral function $\rho(w)$ and has proper support and normalization. By construction, Eq.(27) is consistent with chiral symmetry, although is not necessarily a consequence of it. The result has previously been obtained by several means within the NJL model either using Pauli-Villars regularization^{13,17} on the virtual Compton amplitude (4) or imposing a transverse cut-off¹⁶ upon the quark-target amplitude (5). Within the present approach it can be shown that the forward Compton scattering amplitude (4) in the Bjorken limit yields the finite result presented here²⁰ based on Eq.(5). To really appreciate this point, let us mention that Eq.(27) disagrees with other NJL calculations, due to the use of different regularizations. If Eq.(4) is used with a four-dimensional cut-off¹¹ or Eq.(5) is used with Lepage-Brodsky regularization¹⁶, different shapes for the quark distributions are obtained. The null-plane¹², NJL model¹¹ and spectator model¹⁵ calculations also produce different results. In all cases, the use of momentum dependent form factors or non-gauge invariant regularizations make the connection between Eq.(4) and Eq.(5) doubtful and, furthermore, spoil normalization. The results based on a quark loop with momentum dependent quark masses^{18,19} seem to produce a non-constant distribution.

5 QCD evolution

Eq.(27) can also be understood²³ in terms of phase space arguments and point couplings (i.e., constant matrix elements) which for N constituents gives

$$xV(x, Q_0^2) = N(N-1)x(1-x)^{N-2} \quad \langle xV \rangle \equiv \int_0^1 dx V(x, Q_0^2)x = 1 \quad (28)$$

Of course, it is tempting to look at the nucleon case $N = 3$, although we obviously expect mass corrections to be more important both in the momentum dependence of the $Nqqq$ matrix element and in the phase space, and, in addition we do not have as much theoretical support as in the pion case. As usual,

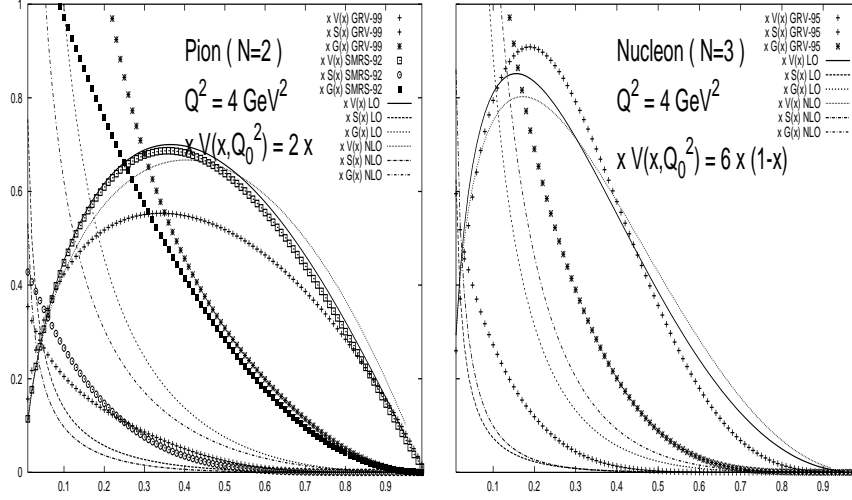


Figure 1. Valence, gluon and sea distributions in the pion (left) and the nucleon (right) at $Q^2 = 4\text{GeV}^2$ compared with phenomenological analysis for the pion SMRS92⁷, GRV99⁸ and for the nucleon GRV95³. We take $\langle xV \rangle_\pi = 0.47$ and $\langle xV \rangle_N = 0.40$ at $Q^2 = 4\text{GeV}^2$. The sea and gluon distributions fulfill $S(x, Q_0^2) = G(x, Q_0^2) = 0$.

we evolve Eq. (28) at LO and NLO exactly implementing the group properties mentioned after Eq.(3) and employed to look for low energy unitarity limits²². Assuming vanishing initial sea and gluon distributions, the result for the dynamically generated valence, sea and gluon distributions for a hadron with $N = 2$ and $N = 3$ constituents compared with known parameterizations for the pion^{7,8} and the nucleon³ respectively can be seen in Fig (1). The agreement with the phenomenological valence distributions is good. Actually, since normalization of the valence distributions is preserved by evolution and Q_0^2 is determined by constraining the valence momentum fraction at $Q^2 = 4\text{GeV}^2$ to the phenomenological ones, $\langle xV \rangle_\pi = 0.47$ and $\langle xV \rangle_N = 0.40$ the bulk of the distribution is reproduced. Sea and gluon distributions do not obey such strong constraints and provide more information regarding the quality of a model. As we see, they are not as good. Nevertheless, a more definite statement might be made if uncertainties in the distributions were considered on the QCD evolution side²⁴. The estimate of systematic errors in the model, i.e. corrections to the initial condition, remains yet a difficult and open problem.

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References

1. R. G. Roberts, *The Structure of the Proton*, Cambridge Monographs on Mathematical Physics, 1990.
2. G. Altarelli and G. Parisi, *Nucl. Phys. B* **126** (1977) 298.
3. M. Glück, E. Reya and A. Vogt, *Z. Phys. C* **67** (1995) 433.
4. A. D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, *Eur. Phys. J. C* **4** (1998) 463.
5. M. Glück, E. Reya and A. Vogt, *Eur. Phys. J. C* **5** (1998) 461,
6. H. L. Lai *et al.*, *Eur. Phys. J. C* **12** (2000) 375.
7. P.J. Sutton, A.D. Martin, R. G. Roberts and W.J. Stirling, *Phys. Rev. D* **45** (1992) 2349;
8. M. Glück, E. Reya, I. Schienbein, *Eur. Phys. J. C* **10** (1999) 313.
9. R. L. Jaffe, *Relativistic Dynamics and Quark Nuclear Physics*, proceedings of the Los Alamos School, 1985, edited by M. B. Johnson and A. Picklesimer (Wiley, New York, 1986).
10. C. Best *et al.* *Phys. Rev. D* **56** (1997) 2743.
11. T. Shigetani, K. Suzuki and H. Toki, *Phys. Lett. B* **308** (1993) 383.
12. T. Frederico and G. A. Miller, *Phys. Rev. D* **50** (1994) 210.
13. R. M. Davidson and E. Ruiz Arriola, *Phys. Lett. B* **359** (1995) 273.
14. C.M. Shakin, Wei-Dong Sun, *Phys. Rev. C* **51** (1995) 2171.
15. R. Jakob, P.J. Mulders, J. Rodrigues, *Nucl. Phys. A* **626** (1997) 937.
16. W. Bentz, T. Hama, T. Matsuki and K. Yazaki, *Nucl. Phys. A* **651** (1999) 143.
17. H. Weigel, E. Ruiz Arriola and L. Gamberg *Nucl. Phys. B* **560** (1999)
18. A.E. Dorokhov and L. Tomio, *Phys. Rev. D* **62** (2000) 014016.
19. M.B. Hecht, C.D. Roberts and S.M. Schmidt, *Phys. Rev. C* **63** (2001) 025213.
20. E. Ruiz Arriola, to be submitted.
21. P. West and R. Delbourgo, *J. Phys. A* **10** (1977). See also R. Delbourgo, *Nuovo Cimento A* **49** (1979) and *hep-th/9903180* and references therein.
22. E. Ruiz Arriola, *Nucl. Phys. A* **641** (1998) 461.
23. E. Ruiz Arriola, Talk given at Bled, Slovenia, 1999. *hep-ph/9910382*
24. K. Nagano, Talk at DIS2001. <http://dis2001.bo.infn.it/wg/sfwg.html>.